Sanjay Ghodawat University, Kolhapur

2020-21

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EXM/P/09/00

Year and Program: SY M.Sc. | **School of Science** | **Department: Mathematics**

Course Code: MTS605 | **Course Title: Field Theory** | **Semester :III**

Day : | **End Semester Examination** | **Time : 2 Hrs.**

Date : | **Max Marks : 50**

Instructions: 1) All Questions are compulsory.
2) Figures to the right indicate full marks.

		Marks	Bloom's Level	CO
Q.1	**Answer the following (Any Two)**			
a)	If L is a finite extension of K and if K is a finite extension of F then prove that L is a finite extension of F.	8	L3	CO1
b)	Prove that an element $a \in K$ is algebraic over F of degree m if and only if $[F(a):F]=m$	8	L4	CO1
c)	If F is a field of characteristic zero and if a and b are algebraic over F then prove that there exists $c \in F(a,b)$ such that $F(a,b)=F(c)$	8	L4	CO2
Q.2	**Answer the following (Any Two)**			
a)	Prove that a field F of characteristic $p \neq 0$ is perfect if and only if for each $a \in F$ there exists $b \in F$ such that $a=b^p$	8	L2	CO2
b)	Let K be a finite extension of F and T be a subfield of K which contains F, then prove that T is normal extension of F if and only if $\sigma(T) \subseteq T \quad \forall \sigma \in G(K,F)$	8	L1	CO3
c)	If F is finite field and α, β are non-zero elements of F then prove that there exists a,b in F such that $1+\alpha a^2+\beta b^2=0$	8	L4	CO3
Q.3	**Answer the following (any two) –**			
a)	Let G be a finite abelian group with the property that the relation $x^n=e$ has n solutions in G for every n, then prove that G is cyclic group.	9	L1	CO4

b)	i) Prove that $a \in F_p$ is primitive n[th] root of unity if and only if $p \equiv 1 (\text{mod}\, n)$	4	L3	CO4
	ii) Let $\alpha \in R$ satisfies an irreducible polynomial of degree n, if n is not a power of two then prove that α is not constructible.	5	L4	CO5
c)	If $p(x) \in F[x]$ is solvable by radicals of F then prove that Galois group of $p(x)$ over F is solvable	9	L4	CO5

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